

# How Public Can a Fact Be?

## Completeness forbids perfect records of charges in bounded regions

Authors to be added

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### Abstract

Quantum Darwinism explains objectivity through redundancy: a fact is objective when many disjoint fragments of the world hold copies of it. We ask how public a fact can be, and whether publicity can be perfect, with zero-error copies in arbitrarily many disjoint bounded fragments. Classically, Gauss’s law makes charge perfectly public: every closed surface around a charge measures it exactly. We show that (i) the facts that every fragment of a commuting family reads with zero error in every state are exactly the projections in the intersection of the fragment algebras, which is abelian and central; (ii) an exact higher-form symmetry makes a charge perfectly public, with zero-error redundancy at least a homological packing number, and equal to it in the toric code; (iii) with dynamical charged matter, Reeh–Schlieder faithfulness and local normality, no bounded region, not even one containing the charge, reads a gauge charge with zero error, and no finite-time local protocol can create such a record. Perfect records exist only at spatial infinity, and the limits  $\delta \rightarrow 0$  and  $r \rightarrow \infty$  do not commute. Granting completeness of the charge spectrum, this is a conditional, effective-field-theory-level “no perfect publicity” counterpart of the Harlow–Ooguri theorem, which we read as “no perfect privacy”. For  $\delta > 0$ , redundancy grows roughly as  $r/(\zeta \ln r)$  for gapped carriers, at least as  $\ln r$  for massless carriers and at least as  $\sqrt{r/\lambda}$  for mass, and stays  $O(1)$  in a Higgs phase. Finally, laws are not records: in the Marolf–Maxfield baby-universe model no radiation fragment holds any information about the coarse law, and infinitely many qubit fragments recover only 0.41% of it at  $S_0 = 2$ .

## 1 Introduction

Why do different observers agree about the world? Quantum Darwinism answers that the environment does not merely decohere a system; it copies selected information about it many times over [1–4]. Observers who intercept different, disjoint fragments of the environment then learn the same fact without disturbing it or each other. The figure of merit is the redundancy  $R_\delta$ , the number of disjoint fragments that each hold at least a fraction  $1 - \delta$  of the information about the fact [3]. Redundancies can be enormous [5]; the structure of such states is well understood [6–8], and objectivity has an operator-algebraic formulation in which redundantly recorded facts sit in centres shared by the fragment algebras [9, 10].

This paper asks how far publicity can go. How public can a fact be, and can publicity be *perfect*: zero error ( $\delta = 0$ ) in arbitrarily many disjoint, bounded fragments? Conserved charges are the natural candidates. Gauss’s law writes the charge enclosed by any closed surface into the electric flux through it, so, classically, each of infinitely many nested, disjoint shells around a charge reads it exactly. The laws of physics, the same for every one, are another candidate.

There is a well-known statement on the opposite

side, about *privacy*. Quantum gravity is believed to have no global symmetries [12–14]. In the language of records, this says that no conserved charge can be perfectly private. In algebraic quantum field theory (AQFT), a charge invisible from the causal complement of a bounded region is a localizable superselection charge in the sense of Doplicher, Haag and Roberts (DHR), and such charges are exactly those of a compact global symmetry group [16]; in a holographic code, an exactly additive charge cannot be hidden from every boundary cell [13]. [Proposition 2](#) makes this reading precise.

Our main result is the mirror image: *completeness of the charge spectrum forbids perfect publicity in bounded regions*. Specifically:

- (i) The facts that every fragment of a commuting family reads with zero error in every state are exactly the projections in  $\bigcap_k \mathcal{A}_k$ , which is abelian, central and immune to fragment-local operations ([Theorem 1](#)).
- (ii) An exact  $p$ -form symmetry with  $p \geq 1$  makes a charge perfectly public on every fragment containing a cycle that links it; in the toric code each fragment either reads the charge perfectly or learns nothing ([Theorem 2](#)).
- (iii) With dynamical charge carriers, faithful (vacuum-

like) states and locally normal charged states, no bounded region reads a gauge charge with zero error, not even a region containing the charge, and no local protocol running for a finite time can create such a record (Theorem 3). The charge is perfectly readable only at infinity, and  $\lim_{r \rightarrow \infty} \lim_{\delta \rightarrow 0} R_\delta(r) = 0$  while  $\lim_{\delta \rightarrow 0} \lim_{r \rightarrow \infty} R_\delta(r) = \infty$  (Theorem 3 and Section 5). Local normality is exactly where dynamical charges enter (Proposition 1), so completeness forbids perfect publicity (Corollary 1; conditional, at the level of effective field theory).

- (iv) For  $\delta > 0$ , redundancy is unbounded outside a Higgs phase, with a growth law set by the carriers (Section 5); and the laws of physics are not records (Section 6).

In one sentence: near-perfect records of a charge are everywhere, but perfect ones exist only at infinity, and no finite region can acquire one in finite time.

**Scope and status of claims.** The setting is quantum field theory on a fixed background, possibly the effective field theory of a gravitating theory; the quantum-gravity statements are conditional on the completeness hypothesis. “Theorem”, “Proposition” and “we show” are used only for statements derived from explicitly stated hypotheses. Unmarked results and numbers were obtained or re-derived in an independent verification analysis. Results marked with a dagger ( $^\dagger$ ) come from a single exploratory analysis that has not yet been independently checked, and are preliminary. Conjectures and heuristics are labelled as such.

**Relation to prior work.** The ingredients are standard: the Reeh–Schlieder theorem [17]; the DHR–Doplicher–Roberts analysis of superselection sectors [16]; the measurability of Gauss-law charges at spacelike infinity [18, 19]; the equivalence, in many gauge theories, of completeness and the absence of (possibly non-invertible) 1-form symmetries [21–24]; and the observation that operators reconstructable on both sides of a bipartition are central [11], of which Theorem 1 is the zero-error, many-fragment version (see also [9, 10]). What is new, as far as we know, is the formulation as statements about redundancy and records: the zero-error no-go in bounded regions and its finite-time strengthening, the order of limits, growth laws for strictly disjoint readers, and the application to baby-universe  $\alpha$ -parameters. A literature search combining the Reeh–Schlieder theorem with quantum Darwinism or objectivity found no statement of this kind.

## 2 Setting and definitions

**Facts and fragments.** A *fact* is a classical random variable  $X$  with values  $x$  in a finite set  $\mathcal{X}$ ,  $|\mathcal{X}| \geq 2$ , and priors  $p_x > 0$ ; when  $X = x$  the world is in the state  $\omega_x$ . A *fragment*  $F$  is a subsystem with a von Neumann algebra  $\mathcal{A}(F)$ . In finite dimensions  $\mathcal{A}(F)$  is a unital  $*$ -subalgebra of  $B(\mathcal{H})$ . In quantum field theory  $F$  is a bounded open region of a Cauchy surface with nonempty causal complement  $F'$ , and  $\mathcal{A}(F)$  is the weak closure of the local  $C^*$ -algebra  $\mathfrak{A}(F)$  in a representation in which the relevant states are normal (fixed in Section 4.2). Disjoint fragments have commuting algebras. The information a fragment holds about  $X$  is the Holevo quantity

$$\chi(X:F) = \sum_x p_x S(\omega_x|_F \| \bar{\omega}|_F), \quad (1)$$

with  $\bar{\omega} = \sum_x p_x \omega_x$  and  $S(\cdot \| \cdot)$  Araki’s relative entropy. It obeys  $0 \leq \chi \leq H(X)$ , cannot increase under channels, and  $\chi(X:F) \leq \chi(X:G)$  whenever  $\mathcal{A}(F) \subset \mathcal{A}(G)$ . Information is in nats.

**Definition 1** (Redundancy [3]).  $F$  is a  $\delta$ -reader of  $X$  if  $\chi(X:F) \geq (1 - \delta)H(X)$ . For a family  $\mathfrak{F}$  of admissible fragments,  $R_\delta(X; \mathfrak{F})$  is the largest number of pairwise disjoint  $\delta$ -readers in  $\mathfrak{F}$ . For a fact carried by an object in a core region  $K$  of radius  $r_0$ ,  $R_\delta(r)$  denotes  $R_\delta$  over bounded fragments inside the ball  $B_r$  around it.

**Definition 2** (Zero-error reading).  $F$  reads  $X$  with zero error if there is a POVM  $\{E_x\} \subset \mathcal{A}(F)$  with  $\omega_x(E_x) = 1$  for all  $x$ ;  $R_0$  is the largest number of disjoint zero-error readers. Given a projection-valued measure (PVM)  $\{P_x\}$  that defines the fact,  $F$  reads it *robustly* if  $\omega(E_x) = 1$  for every normal state  $\omega$  with  $\omega(P_x) = 1$ , for all  $x$ .

**Lemma 1** (Zero error means orthogonal supports). *Let the  $\omega_x|_F$  be normal with support projections  $s_x \in \mathcal{A}(F)$ .  $F$  reads  $X$  with zero error if and only if  $s_x s_y = 0$  for all  $x \neq y$ , and then  $\chi(X:F) = H(X)$ . In finite dimensions the converse also holds, so  $R_0 = R_{\delta=0}$ .*

*Proof.* If the supports are orthogonal,  $E_x = s_x$  (with  $1 - \sum_x s_x$  added to any one element) works. Conversely,  $\omega_x(E_x) = 1$  gives  $E_x s_x = s_x$ , and for  $y \neq x$ ,  $\omega_y(E_x) \leq \omega_y(1 - E_y) = 0$  gives  $E_x s_y = 0$ ; hence  $s_y s_x = s_y E_x s_x = 0$ . With orthogonal supports each term of (1) equals  $-\ln p_x$ , so  $\chi = H(X)$ . In finite dimensions  $\chi = H(X)$  forces orthogonal supports (the equality condition of the Holevo bound).  $\square$

**Definition 3** (Perfect publicity, perfect privacy, records at infinity). Let  $X$  be a charge carried by an object in a bounded region  $K$ . (a)  $X$  is *perfectly public* if  $R_0 = \infty$  over bounded fragments, and *robustly* so if the readers read it robustly. (b)  $X$  is *perfectly private* if  $\omega_x|_{\mathfrak{A}(K')} = \omega_{x'}|_{\mathfrak{A}(K')}$  for all  $x, x'$ : no observer outside  $K$  learns anything. (c)  $X$  is *perfectly readable at infinity* if, for every bounded  $\tilde{K} \supset K$ ,

the algebra of the causal complement  $\tilde{K}'$  reads  $X$  with zero error.

**Gauss's law and flux readers.** Let a static charge  $q$  sit in  $K$ . For a smooth  $f$  equal to 1 on  $K$  and 0 outside a shell  $F$  around it, Gauss's law gives the operator identity

$$\Phi_f := - \int d^3x \mathbf{E} \cdot \nabla f = \int d^3x f \rho, \quad (2)$$

where  $\rho$  is the charge density of the source plus any dynamical matter. The left side is supported where  $\nabla f \neq 0$ , so  $\Phi_f \in \mathcal{A}(F)$ ; the right side shows what the shell reads: the smeared enclosed charge, including every charge fluctuation inside the shell. Without dynamical charged matter,  $\Phi_f = q$  exactly in the charge- $q$  sector, every enclosing shell is a zero-error reader, and  $R_0 = \infty$ : the classical picture. The question is what survives when charged matter fluctuates.

### 3 Robust perfect publicity requires a shared centre

**Theorem 1** (Universal public algebra). *Let  $\mathfrak{F} = \{\mathcal{A}_1, \dots, \mathcal{A}_n\}$ ,  $n \geq 2$ , be pairwise commuting von Neumann algebras on  $\mathcal{H}$ , and let  $\{P_x\}$  be a PVM.*

- (0) *Fragment  $k$  reads  $\{P_x\}$  robustly iff  $P_x \in \mathcal{A}_k$  for all  $x$ , and the reading POVM is then  $\{P_x\}$ . Hence the facts that every fragment of  $\mathfrak{F}$  reads robustly are exactly the PVMs in  $\mathcal{P}(\mathfrak{F}) := \bigcap_k \mathcal{A}_k$ .*
- (1)  *$\mathcal{P}(\mathfrak{F})$  is abelian.*
- (2)  *$\mathcal{P}(\mathfrak{F}) \subset Z(\mathcal{A}_k)$  for every  $k$ , and  $\mathcal{P}(\mathfrak{F}) \subset Z(\bigvee_k \mathcal{A}_k)$ .*
- (3) *For every instrument with Kraus operators  $\{K_i\} \subset \mathcal{A}_j$  (any  $j$ ),  $\sum_i K_i^\dagger X K_i = X$  for all  $X \in \mathcal{P}(\mathfrak{F})$ : no fragment-local operation disturbs or erases a robustly public fact.*
- (4) *If  $\bigvee_k \mathcal{A}_k$  is a factor, then  $\mathcal{P}(\mathfrak{F}) = \mathbb{C}\mathbf{1}$  and no nontrivial fact is robustly public.*

*Proof.* (0) If  $P_x \in \mathcal{A}_k$ , take  $E_x = P_x$ . Conversely,  $\omega(E_x) = 1$  on vector states in  $P_x \mathcal{H}$  gives  $E_x P_x = P_x$ ; then  $\sum_{y \neq x} P_x E_y P_x = P_x - P_x E_x P_x = 0$ , so positivity gives  $E_y P_x = 0$  for  $y \neq x$ , and  $E_x = E_x \sum_y P_y = E_x P_x = P_x \in \mathcal{A}_k$ . (1) For  $X, Y \in \mathcal{P}$ :  $X \in \mathcal{A}_1$  and  $Y \in \mathcal{A}_2 \subset \mathcal{A}'_1$ , so  $XY = YX$ . (2)  $X \in \mathcal{A}_k \cap \mathcal{A}_j \subset \mathcal{A}_k \cap \mathcal{A}'_k = Z(\mathcal{A}_k)$  for  $j \neq k$ ; and  $X$  lies in  $\bigvee_i \mathcal{A}_i$  and commutes with every  $\mathcal{A}_i$ . (3)  $X \in \mathcal{A}_i$  for some  $i \neq j$ , so  $[K_i, X] = 0$  and  $\sum_i K_i^\dagger X K_i = X$ . (4) The centre of a factor is  $\mathbb{C}\mathbf{1}$ .  $\square$

Robust perfect publicity is therefore kinematic: the fact must be a superselection label in every fragment's centre, and it then carries the strongest persistence guarantee possible, item (3). Being central in the *total* algebra is not enough (Section 6), and a fact can be read with zero error in particular states without being robustly

public; Section 4 treats that state-dependent question for charges. The bipartite case of Theorem 1 is the centrality of operators reconstructable on both sides of a cut [11]; the redundancy notion of [10] uses the same intersection of centres.

## 4 Perfect publicity of charges

### 4.1 Exact higher-form symmetries

Consider a theory on a  $d$ -dimensional spatial manifold  $M$  with an exact  $p$ -form symmetry,  $p \geq 1$ , with abelian group  $G$ . Its operators  $U_g(\Sigma)$  live on closed  $(d-p)$ -dimensional submanifolds  $\Sigma$  and are topological, depending on  $\Sigma$  only through its homology class [20]; for the electric 1-form symmetry of pure Maxwell theory in  $d = 3$ ,  $U_a(\Sigma) = \exp(i\alpha \oint_\Sigma \mathbf{E} \cdot d\mathbf{A})$ . Let a charge  $q$  (a character  $\chi_q$  of  $G$ ) sit in a compact  $K$ . In the charge- $q$  sector  $U_g(\Sigma) = \chi_q(g)\mathbf{1}$  for every  $\Sigma \subset M \setminus K$  in the class  $[\Sigma_0] \in H_{d-p}(M \setminus K)$  that links  $K$ . For a family  $\mathfrak{F}$  of disjoint fragments, the *homological packing number* is

$$v(\mathfrak{F}) = \#\{F \in \mathfrak{F} : F \supset \text{a representative of } [\Sigma_0]\}. \quad (3)$$

**Theorem 2** (Homological redundancy). *Under these assumptions:*

- (a) *every fragment counted in  $v(\mathfrak{F})$  contains the charge projections  $P_q$  (the joint spectral projections of  $\{U_g(\Sigma_F)\}_{g \in G}$  for a representative  $\Sigma_F \subset F$ ), which are the same operators for every representative. Hence  $R_0 \geq v(\mathfrak{F})$ , and these fragments read  $q$  robustly, with the  $P_q$  in the intersection of their algebras as in Theorem 1;*
- (b) *for  $p = 0$  the only representative is all of space, so  $v \leq 1$ ;*
- (c) *in the  $\mathbb{Z}_N$  toric code with a static electric charge at a vertex  $v_0$  (its partner outside the region considered), a fragment's reduced state depends on the charge iff the fragment contains a closed dual loop enclosing  $v_0$ . Hence  $R_\delta = v$  for every  $\delta \in [0, 1)$ : each fragment either reads the charge perfectly or learns nothing.*

*Proof.* (a)  $U_g(\Sigma_F) \in \mathcal{A}(F)$  because  $\Sigma_F \subset F$ ; by topological invariance it acts as  $\chi_q(g)$  in the charge- $q$  sector whatever the representative, so its joint spectral projections are the sector projections, and they read  $q$  with zero error in every state. (b) is immediate. (c) We write it for  $\mathbb{Z}_2$ ;  $\mathbb{Z}_N$  is identical with generalized Pauli operators. The state is a stabilizer state with generators  $A_v = \prod_{l \ni v} X_l$  and plaquette terms  $B_p$ , and  $A_{v_0} = (-1)^q$ . A fragment's reduced state is fixed by the stabilizer elements supported in it, with their signs. An  $X$ -type element is  $\prod_{v \in S} A_v = \prod_{l \in \delta S} X_l$  for a vertex set  $S$  with coboundary  $\delta S$ ; its sign is  $(-1)^q$  if  $v_0 \in S$  (the partner not in  $S$ ) and  $q$ -independent otherwise, and  $Z$ -type elements do not depend on  $q$ . So  $\rho_F$  depends on  $q$  iff  $F \supset \delta S$

for some such  $S \ni v_0$ , i.e. iff  $F$  contains a closed dual loop enclosing  $v_0$ , and then measuring  $\prod_{l \in \delta S} X_l$  reads  $q$  perfectly; otherwise  $\chi(q:F) = 0$ .  $\square$

A ball of radius  $r$  in the continuum contains infinitely many disjoint shells, so an exact 1-form symmetry makes the charge robustly perfectly public; [Theorem 2](#) is the shell-family instance of [Theorem 1](#). We state it for invertible symmetries; the argument uses only that the operators are topological and separate the sectors by scalars, so it may extend to non-invertible ones, which we have not checked.

## 4.2 Dynamical charges: no perfect publicity in bounded regions

Now let the charge be carried by dynamical matter. We work in AQFT on a globally hyperbolic spacetime: a net  $O \mapsto \mathfrak{A}(O)$  of  $C^*$ -algebras satisfying isotony, locality and primitive causality (the time-slice property), with vacuum representation  $\pi_0$  and vacuum vector  $\Omega$  [17]. For bounded  $F$  write  $\mathcal{A}_0(F) = \pi_0(\mathfrak{A}(F))''$ . A charge fact is a family of states  $\{\omega_q\}_{q \in \mathcal{Q}}$  in which an object localized in a bounded  $K$  carries charge  $q$ . We assume:

- (F) *Faithfulness*. For every bounded  $F$  with  $F' \neq \emptyset$ , all but at most one of the  $\omega_q|_F$  are faithful on  $\mathcal{A}_0(F)$ .
- (N) *Local normality*. For every bounded  $F$ , each  $\omega_q|_{\mathfrak{A}(F)}$  is normal with respect to  $\pi_0$ , i.e. extends to a normal state on  $\mathcal{A}_0(F)$ .

By the Reeh–Schlieder theorem the vacuum is faithful on  $\mathcal{A}_0(F)$  for every bounded  $F$  with  $F' \neq \emptyset$ , including regions that contain  $K$ , so (F) holds whenever the vacuum is one value of the fact (“charge  $q$  present or absent”). It also holds for vectors of bounded energy support and for  $W\psi$  with  $\psi$  cyclic for every local algebra (such as the vacuum) and  $W$  a unitary localized in a bounded region; for ground states with a static source it is an assumption, plausibly accessible through the timelike tube theorem [27]. Under (N) all  $\omega_q$  are normal states on the one algebra  $\mathcal{A}_0(F)$ , which we take as the fragment algebra.

**Lemma 2** (Faithful versus normal). *If  $\varphi$  is a faithful and  $\omega$  a normal state on a von Neumann algebra, their supports are not orthogonal and their fidelity is nonzero.*

*Proof.*  $s(\varphi) = \mathbf{1}$ , so  $s(\omega)s(\varphi) = s(\omega) \neq 0$ ; normal states have zero fidelity iff they are mutually singular (Fuchs–van de Graaf), iff their supports are orthogonal.  $\square$

**Theorem 3** (No perfect publicity in bounded regions). *Assume (F) and (N). Then:*

- (a) *No bounded fragment distinguishes any two charges  $q \neq q'$  with zero error, including fragments that contain the charge. Hence  $R_0 = 0$  over bounded fragments.*

(b) *This persists in time and under local operations. After unitary evolution for any finite time, and after any finite sequence of operations localized in bounded regions (with ancillas prepared independently of  $q$ , and post-selection on outcomes of nonzero probability), no bounded region holds a zero-error record of  $q$ . Exactness cannot be imported from infinity in finite time.*

- (c) *For a binary fact  $q \in \{q_0, q_1\}$  and every finite  $r$  there is  $\delta_r > 0$  with  $R_\delta(r) = 0$  for all  $\delta < \delta_r$ ; hence  $\lim_{\delta \rightarrow 0} R_\delta(r) = 0$  for every finite  $r$ .*

*Proof.* (a) Suppose a POVM  $\{E_x\} \subset \mathcal{A}_0(F)$  has  $\omega_q(E_q) = \omega_{q'}(E_{q'}) = 1$ . Then  $\omega_{q'}(E_q) \leq \omega_{q'}(\mathbf{1} - E_{q'}) = 0$ . By (F) one of the two states is faithful; if it is  $\omega_{q'}$  then  $E_q = 0$ , contradicting  $\omega_q(E_q) = 1$ , and the other case is symmetric. Reeh–Schlieder applies to every bounded region with nonempty causal complement, so regions containing  $K$  are included.

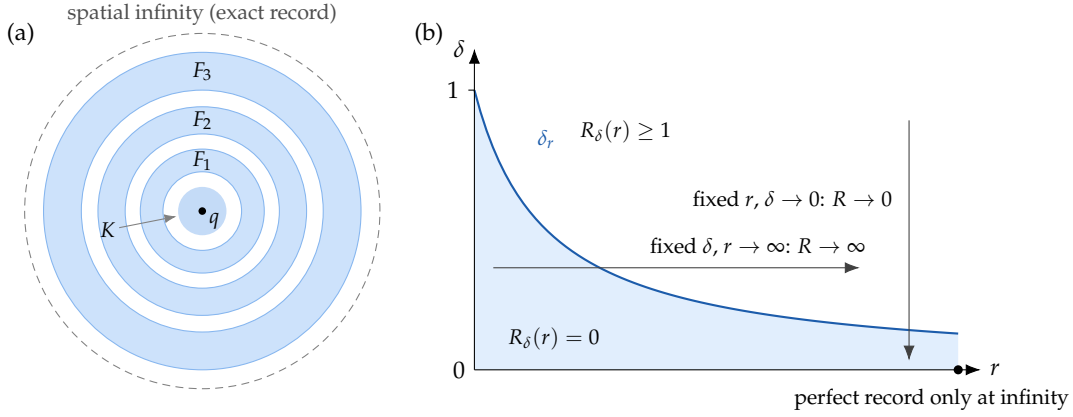
(b) By primitive causality, the Heisenberg-evolved algebra of a bounded region after a finite time  $T$ , together with the operations performed in bounded regions during  $[0, T]$ , lies in  $\mathcal{A}_0(G) \otimes B(\mathcal{H}_{\text{anc}})$  for a bounded region  $G$  of the initial Cauchy surface and the ancillas. The initial states  $\omega_q|_G \otimes \sigma$  have fidelity  $F(\omega_q|_G, \omega_{q'}|_G) > 0$  by (F), (N) and [Lemma 2](#). Fidelity does not decrease under channels or restriction, whereas a zero-error record needs zero fidelity. For post-selection with a Kraus operator  $K$ , faithfulness of  $\omega_{q'}$  makes the support of the post-selected  $q'$ -state equal to the range projection of  $K(\mathbf{1} \otimes \sigma^{1/2})$ , which contains the support of the post-selected  $q$ -state; the two are never orthogonal.

(c) Every fragment inside  $B_r$  has  $\mathcal{A}_0(F) \subset \mathcal{A}_0(B_r)$ , so  $\chi(X:F) \leq \chi(X:B_r)$ . By Uhlmann’s theorem (valid in the standard form of a von Neumann algebra) the two restricted states have vector representatives with overlap  $F_r := F(\omega_{q_0}|_{B_r}, \omega_{q_1}|_{B_r}) > 0$  ([Lemma 2](#)), and monotonicity bounds  $\chi(X:B_r)$  by the Holevo quantity of two pure states with overlap  $F_r$ , which is strictly less than  $H(X)$ . Take  $\delta_r = 1 - \chi(X:B_r)/H(X)$ .  $\square$

**Records at infinity and the order of limits.** For charges obeying a Gauss law in a Coulomb phase, the electric flux at spacelike infinity is a superselected observable, measurable in the causal complement of any bounded region [18, 19]; the  $\omega_q$  are mutually singular there, so the charge is perfectly readable at infinity. With the growth laws of [Section 5](#), which give  $\lim_{r \rightarrow \infty} R_\delta(r) = \infty$  for every  $\delta > 0$  in every Coulomb-type carrier class, [Theorem 3\(c\)](#) yields

$$\lim_{r \rightarrow \infty} \lim_{\delta \rightarrow 0} R_\delta(r) = 0, \quad \lim_{\delta \rightarrow 0} \lim_{r \rightarrow \infty} R_\delta(r) = \infty. \quad (4)$$

Perfect publicity is the second, non-commuting limit, attained only at infinity ([Fig. 1](#)). The Higgs phase is the exception: the charge is screened, and  $R_\delta = O(1)$ .



**Figure 1:** (a) A charge  $q$  in a core  $K$  with disjoint shells  $F_1, F_2, F_3, \dots$ , each a candidate reader of the enclosed flux (2). (b) Schematic of the order of limits (4), not to scale. Below the curve  $\delta_r$  no fragment inside  $B_r$  is a  $\delta$ -reader (Theorem 3(c));  $\delta_r > 0$  for every finite  $r$ , and  $\delta_r \rightarrow 0$  as  $r \rightarrow \infty$  because  $R_\delta(r) \rightarrow \infty$  for every  $\delta > 0$  (Section 5). Zero-error records are reached only when  $r \rightarrow \infty$  is taken first.

### 4.3 Where completeness enters

**Proposition 1** (Local normality from dynamical charges; physical derivation). *Suppose a dynamical charge  $-q$  exists, created by an operator dressed with a string (or cone) that extends to infinity. Then (N) holds for bounded  $F$ :*

1. Bring a compensating charge  $-q$  from infinity along a cone  $C \subset F'$  to a point just outside  $F$ , in the unbounded component of its complement. The operator is localized in  $C$  and  $\mathfrak{A}(C) \subset \mathfrak{A}(F)'$ , so the state on  $\mathfrak{A}(F)$  is unchanged.
2. Replace the string dressing of  $-q$  by its Coulomb dressing. The two electric fields differ by a divergence-free, square-integrable field, implemented by a Weyl operator  $W$ ; the resulting state  $\omega'$  (charge  $q$  in  $K$ , a Coulomb-dressed  $-q$  nearby) is in the vacuum sector.
3. Restricted to  $F$ , the field implemented by  $W$  has zero flux through every closed surface in  $F$ , so it extends to a compactly supported divergence-free field, and  $W$  can be replaced by a Weyl operator  $\tilde{W}$  localized near  $F$  with the same action on  $\mathfrak{A}(F)$ .

Hence  $\omega_q|_{\mathfrak{A}(F)} = (\omega' \circ \text{Ad } \tilde{W}^{-1})|_{\mathfrak{A}(F)}$ , which is  $\pi_0$ -normal.

Without dynamical charges step 1 is impossible: the flux  $\Phi_f$  of (2) is then the number  $q$  in the charge- $q$  sector and 0 in the vacuum sector, the sectors are disjoint on the shell algebras, (N) fails, and Theorem 2 applies. This is the dichotomy: charges that dynamical carriers can end have  $R_0 = 0$  in bounded regions; charges carried by an unendable topological flux (an exact 1-form symmetry) have  $R_0 \geq \nu$ . Proposition 1 is rigorous at the level of free-field Weyl operators; in continuum QED, whose charged sectors have infrared subtleties, we regard (N) as an assumption. A theory with locally superselected charges but no exact 1-form symmetry would evade

Theorem 3; we know of none.

**Corollary 1** (Completeness forbids perfect publicity; conditional, EFT level). *Assume completeness: every Wilson and 't Hooft line can end on a dynamical object [12, 14, 15], so there is no exact, possibly non-invertible, electric or magnetic 1-form symmetry [21–23]. Assume (F), and (N) as derived in Proposition 1. Then no gauge charge has a zero-error record in any bounded region, in vacuum-like states or in any state reached from them by local operations in finite time. Exact records exist only at infinity.*

The direction needed, completeness  $\Rightarrow$  no exact 1-form symmetry, is elementary: if a line can end, a topological surface linking it can be slid off the endpoint and acts trivially on the line. The converse is proved in [22]; for non-abelian finite gauge groups the equivalence requires non-invertible operators [21, 22]. In AQFT, completeness is the maximality of the local algebras, equivalent to the absence of generalized symmetries [23]; Proposition 1 supplies the further step to (N). The statement is at the level of effective field theory. McNamara and Vafa argue that consistent quantum gravity has no nontrivial, strictly well-defined bulk operators supported in compact regions [28]; if so, the argument becomes formal, and its conclusion (no exact bounded-region records) only strengthens. Completeness allows very heavy carriers, such as black holes; then  $R_0 = 0$  exactly while  $R_\delta$  can be astronomically large, so the no-go is sharp only at  $\delta = 0$ .

### 4.4 Scope

*Infinity.* Exact records of gauge charges survive only in the algebras of complements of bounded regions, as the asymptotic flux; the asymptotic (ADM) Poincaré charges are the gravitational analogue (Section 5). Closed universes. With compact Cauchy surfaces there is no

| dynamical matter         | $\mu$ | $T$   | $F$   | $\chi / \ln 2$ |
|--------------------------|-------|-------|-------|----------------|
| none (exact 1-form sym.) | –     | 1     | 0     | 1              |
| all sites but the probe  | 0.5–8 | 1     | 0     | 1              |
| all sites                | 0.25  | 0.253 | 0.956 | 0.062          |
|                          | 1     | 0.647 | 0.714 | 0.378          |
|                          | 4     | 0.976 | 0.181 | 0.922          |
|                          | 16    | 0.998 | 0.046 | 0.991          |

**Table 1:** Lattice illustration of [Theorem 3](#): ground states of  $H = -h \sum_l (1 - 2e_l) - J \sum_p B_p - t \sum_l F_l + \mu \sum_v \eta_v$  ( $\mathbb{Z}_2$  electric variables  $e_l$ , plaquette flips  $B_p$ , gauge-invariant hopping and pair creation  $F_l$ , matter parity  $\eta_v$ ) on an open  $3 \times 3$  grid,  $h = J = 1$ ,  $t = 0.5$ , with static probes at the centre and a corner. The fragment is the four links around the central probe;  $T$  and  $F$  are the trace distance and root fidelity between its states with and without the probe charge, and  $\mu$  is the matter gap. When matter can sit at the probe site, the Hamiltonian is stoquastic and connected on a configuration space containing every flux configuration of the four links, so by Perron–Frobenius their marginals have full support for both charges (a lattice analogue of Reeh–Schlieder): the fidelity never vanishes, decaying as  $\approx 0.73/\mu$ . A matter-free probe site is an exact enclosure, a lattice loophole. The enclosure reader (four links plus the probe-site matter) reads the charge with certainty in every case.

asymptotic region and the total gauge charge vanishes, so, under the same hypotheses, no proper region holds a zero-error record of a localized charge, and neither perfect privacy nor perfect publicity occurs (a sketch). *Non-faithful states.* Using the split property one can engineer states in which each of  $n$  disjoint bounded fragments reads the charge with zero error; such states are not energy-analytic and, by [Theorem 3\(b\)](#), cannot be prepared from faithful states by local operations in finite time. [Theorem 3](#) is thus a theorem about the physically distinguished class of faithful states and their finite-time descendants, not about the algebras alone.  $\delta > 0$ . Publicity is unbounded ([Section 5](#)); the no-go concerns exactness only.

*Lattices.* On a lattice (N) is automatic, but sharp enclosures exist: a fragment containing a closed set of links together with all the matter they enclose contains the Gauss-law parity as a central element. [Table 1](#) shows both effects in a  $\mathbb{Z}_2$  gauge theory with  $\mathbb{Z}_2$  matter. With unknown (maximally mixed) dynamical matter, a link flux or an enclosed charge has exactly 3, 4 or 2 disjoint zero-error readers for link, half-link or vertex-cell access, independent of lattice size (certified by explicit packings and matching transversals for  $\mathbb{Z}_2$  and  $\mathbb{Z}_3$  in two and three dimensions). One reader of each type is the whole complement, the lattice stand-in for infinity. The regularization-independent content is “bounded” versus “unbounded”.

## 4.5 The mirror image: no perfect privacy

**Proposition 2** (No perfect privacy). (a) *Within the DHR framework, a perfectly private charge ([Definition 3\(b\)](#)) is a localizable superselection charge, and such sectors are exactly those of a compact global gauge group [\[16\]](#). Conversely, a DHR-localized charge is invisible from the causal complement of its localization region. Perfect privacy is therefore the signature of a global 0-form symmetry, which quantum gravity is believed to forbid [\[12–14\]](#). For Gauss-law charges the complement of every bounded region reads the charge exactly [\[18, 19\]](#).*

(b) *In a quantum error-correcting code with exact complementary recovery and an exactly additive charge  $Q = \sum_j q_j$  that preserves the code, every partition of the boundary into cells  $R_i$  gives  $Q_L = \sum_i F_i$  with commuting  $F_i$  in the cells’ logical algebras, so a charge-changing logical operator is visible to some cell of every partition [\[11, 13\]](#). Quantitatively, for a binary charge label with equal priors, if every cell of an  $n$ -cell partition holds at most  $\lambda < \frac{1}{2}$  nats about it, then*

$$\Delta Q \leq n \sqrt{8 v_{\max} \lambda / (1 - 2\lambda)}, \quad (5)$$

where  $v_{\max}$  bounds the variances of the cell charges.

*Proof of (5).* For two distributions with equal priors, means differing by  $\Delta$  and variances  $v_0, v_1$ , the mutual information obeys  $I \geq \Delta^2 / (4(v_0 + v_1) + 2\Delta^2)$  (bound the Jensen–Shannon divergence by the triangular discrimination and apply Cauchy–Schwarz). The cell mean shifts add up to  $\Delta Q$ , so some cell has shift  $\geq \Delta Q/n$ , and measuring its charge gives  $\lambda \geq (\Delta Q/n)^2 / (8v_{\max} + 2(\Delta Q/n)^2)$ .  $\square$

Perfect privacy ( $\lambda = 0$ ) forces  $\Delta Q = 0$ , a record-theoretic form of the Eastin–Knill obstruction and its approximate versions [\[29, 30\]](#). [Proposition 2](#) and [Corollary 1](#) are the two halves of one statement: granting completeness, quantum gravity forbids both perfect privacy and perfect publicity of charges.

## 5 Imperfect publicity: how fast redundancy grows

For  $\delta > 0$  the question becomes quantitative. A shell reader is a smeared flux

$$X_F = \int d^4x \alpha(t) f(\mathbf{x}) O(\mathbf{x}), \quad O \in \{j^0, T^{00}\}, \quad (6)$$

with  $f$  as in [\(2\)](#) and a time window  $\alpha$  supported in the domain of dependence  $D(F)$ , so that  $X_F \in \mathcal{A}(F)$ . By the constraint, the mean of  $X_F$  is shifted by the enclosed charge  $q$  (or energy  $\Delta M$ ), and its noise is the vacuum variance of the smeared density. A Gaussian readout reaches level  $\delta$  iff  $(\Delta\mu/\sigma)^2 \geq 8\kappa(\delta)$ , with  $\kappa = 1.879$ ,

3.956, 6.103 at  $\delta = 0.1, 0.01, 0.001$  (the equal-prior binary Gaussian channel;  $\kappa \simeq \ln(1/\delta)$  asymptotically). In a conformal field theory the spectral densities at frequency  $\Omega$  and wavenumber  $Q$  are  $\rho_{j^0} = c_j Q^2 \theta(\Omega - |Q|)$  and  $\rho_{T^{00}} = \frac{2}{3} c_T Q^4 \theta(\Omega - |Q|)$ , with  $c_j = 1/(48\pi^2)$  per complex scalar ( $1/(12\pi^2)$  per Dirac fermion),  $c_T = 1/(960\pi^2)$  per conformal scalar and  $1/(80\pi^2)$  for the Maxwell field. For Gaussian profiles of width  $w$ , time window  $\tau = \beta w$  and a shell of area  $A \gg w^2$ ,

$$\begin{aligned} \text{Var } Q &= \frac{A c_j \arctan(1/\beta)}{2\pi\beta w^2}, \\ \text{Var } E &= \frac{A c_T [\arctan(1/\beta) - \beta/(1 + \beta^2)]}{6\pi\beta w^4}. \end{aligned} \quad (7)$$

Dimensional analysis fixes the growth laws.  $j^0$  has mass dimension 3 and  $T^{00}$  dimension 4, so the shell noise scales as  $A/w^2$  for charge and  $A/w^4$  for energy. With  $A = 4\pi\rho^2$ , a charge with massless carriers needs  $w \geq x_\delta \rho$ , so disjoint shells grow geometrically and  $R_\delta \propto \ln r$ ; energy needs  $w \geq b_\delta \sqrt{\rho \lambda_{\Delta M}}$  with  $\lambda_{\Delta M} = \hbar/(\Delta M c)$ , so  $R_\delta = \int d\rho/w_{\min} = (2/b_\delta) \sqrt{r/\lambda_{\Delta M}}$ . The constants depend on the reader. Gaussian profiles packed one width apart give  $b_{0.1} = 0.245$  and, for one Dirac fermion,  $x_{0.1} = 0.447$ , but such shells overlap and their windows leave  $D(F)$ . Strictly disjoint readers, with product windows supported in each shell's domain of dependence, give  $b_{0.1} = 1.72$  and  $x_{0.1} = 2.12$ , hence

$$\begin{aligned} R_{0.1}(\Delta M; r) &\geq 1.16 \sqrt{r/\lambda_{\Delta M}}, \\ R_{0.1}(q; r) &\geq 0.878 \ln(r/r_0), \end{aligned} \quad (8)$$

the second for one massless Dirac fermion (at least 24 disjoint readers over twelve decades of radius). These are achievability bounds: non-product windows may do better, and no upper bound is known. The  $\delta$ -dependence enters only through  $\kappa$ :  $R \propto \kappa^{-1/4}$  for mass and  $R \propto 1/\ln(1 + x_1 \sqrt{\kappa})$  for massless charges.

**Carrier classes** (Table 2). *Gapped carriers* with correlation length  $\xi$  cross a shell of width  $w$  somewhere on its area with probability  $\propto (\text{area}/\xi^2) e^{-w/\xi}$ , so  $w_{\min} \simeq \xi \ln(\rho/\xi \varepsilon_\delta)$  and  $R_\delta(r) = \Theta(r/(\xi \ln r))$ . On a lattice this is a theorem: for the toric code with independent errors at rate  $p$  (incoherently created charge pairs), Peierls-type path counting bounds the failure probability of the best annulus reader from above for  $p < 1/36$  and from below for all  $p$ , both exponentially in  $w$ . A minimum-weight-matching reader, packed greedily, gives 11 and 5 disjoint  $\delta = 0.1$  readers within 100 rings at  $p = 0.02$  and  $0.05$ , and its redundancy collapses to  $O(1)$  near  $p \approx 0.10$ , the known decoding threshold [31]<sup>†</sup>. For relativistic carriers of mass  $m$  a scaling argument gives  $R_\delta = \tilde{\Theta}(mr)$ . Massless carriers give (8); scale invariance suggests that  $\ln r$  is also an upper bound, but this is open. In a Higgs phase the condensate screens the charge and  $R_\delta = O(1)$ .

**Gravity's own charges.** The gravitational Gauss law reads the enclosed energy, and every field gravitates, massless ones included, so there is no gap. By the linearized constraint, the ADM-type surface integral over any closed surface in a shell equals the enclosed energy, including the fluctuating energy of every field in the shell; stress-tensor fluctuations do source metric fluctuations [32]. Mass is therefore only square-root public in bounded regions: with photon noise alone, (8) gives at least  $2.0 \times 10^{21}$  disjoint readers for 1 kg at 1 m,  $2.6 \times 10^3$  for 1 eV and 12 for a single 5 GHz photon, in vacuum. Other massless fields add noise and lower the coefficient by an  $O(1)$  factor; the graviton's own contribution is not locally well defined. We have not formulated the analogue of Theorem 3 for Poincaré charges, since local normality of states with different ADM mass requires a treatment of gravitational dressing [33]; we conjecture  $R_0 = 0$  there too, with exact records only at infinity. Preliminary results<sup>†</sup> sharpen the picture. (i) Optimizing over readers “Gauss flux minus an arbitrary shell observable” raises the coefficient to  $R_{0.1} \approx 2.55 \sqrt{r/\lambda_{\Delta M}} = 1.80 \sqrt{r r_g}/l_P$  ( $r_g = 2G\Delta M/c^2$ ), with a matching upper bound within that reader class; arbitrary measurements are open. (ii) Momentum is also square-root public, while angular momentum and the boost charge have a lever arm  $\rho$  and are only logarithmically public: a charge of mass dimension one is square-root public, a dimensionless one logarithmically public. (iii) With conserved matter, the 20 topological operators of the linearized graviton [34] split into 10 that read the enclosed Poincaré charges, 4 that vanish identically on every closed surface and 6 that read only the stress on the surface itself; the non-Poincaré ones record nothing about the interior. (iv) In a thermal bath the flux cannot separate  $\Delta M$  from volume-law energy fluctuations; the cosmic microwave background hides energy records below  $\approx 124 \text{ eV } (\rho/1 \text{ m})^{3/2}$ .

## 6 The laws are not records

The laws of physics are the same for every observer, so they are natural candidates for perfectly public facts. In baby-universe treatments of the gravitational path integral, coupling constants and other “laws” are labelled by  $\alpha$ -parameters [35–37], and the  $\alpha$ -states define superselection sectors for the boundary observables [38].

### 6.1 Graded and ungraded fragments

**Theorem 4** ( $\alpha$ -sectors). *Let  $\mathcal{H} = \bigoplus_\alpha \bigotimes_{k=1}^n \mathcal{H}_{k,\alpha}$  (factorization within each sector,  $n \geq 2$ ), and let each fragment be graded,  $\mathcal{A}_k = \bigoplus_\alpha B(\mathcal{H}_{k,\alpha}) \otimes \mathbf{1}$ , so that it contains the sector projections. Then  $\bigcap_k \mathcal{A}_k = \{\bigoplus_\alpha c_\alpha \mathbf{1}_\alpha\}$ , the  $\alpha$ -algebra:  $\alpha$  is robustly public, and nothing else is. If the fragment algebras share a nontrivial centre inside some sector, the refined label*

| Carriers / charge                             | $R_\delta(r)$                                  | Example ( $\delta = 0.1$ )                                                                                           | Status                                            |
|-----------------------------------------------|------------------------------------------------|----------------------------------------------------------------------------------------------------------------------|---------------------------------------------------|
| none: exact 1-form symmetry                   | $\nu$ (disjoint linking shells)                | toric code: $R_\delta = \nu$ for all $\delta < 1$                                                                    | <a href="#">Theorem 2</a>                         |
| any dynamical carriers                        | $R_0 = 0$                                      | bounded regions, faithful states                                                                                     | <a href="#">Theorem 3</a>                         |
| gapped (length $\xi$ )                        | $\Theta(r/(\xi \ln r))$                        | toric code with i.i.d. errors: 11 readers ( $p = 0.02$ ), 5 ( $p = 0.05$ ) within 100 rings <sup>†</sup>             | lattice: Peierls bounds; continuum: scaling       |
| massless (one Dirac fermion)                  | $\geq 0.878 \ln(r/r_0)$                        | $\geq 24$ over twelve decades                                                                                        | achievability; upper bound open                   |
| Higgs phase                                   | $O(1)$                                         | screened beyond the screening length                                                                                 | standard                                          |
| mass $\Delta M$ (photon noise)                | $\geq 1.16\sqrt{r/\lambda_{\Delta M}}$         | $r = 1$ m: $1 \text{ kg} \geq 2.0 \times 10^{21}$ ; $1 \text{ eV} \geq 2.6 \times 10^3$ ; one 5 GHz photon $\geq 12$ | achievability; upper bound open                   |
| angular momentum $\Delta J \gg \hbar^\dagger$ | $\approx 1.47\sqrt{\Delta J/\hbar} \ln(r/r_0)$ | logarithmic, like a massless-carrier charge                                                                          | scaling argument; numerical constant <sup>†</sup> |
| lattice, unknown matter                       | $R_0 = 3/4/2$                                  | links / half-links / cells, size-independent                                                                         | exact certificates                                |

**Table 2:** How fast redundancy grows with the radius  $r$  of the region, by charge carrier (or, for gravity, by charge). “Achievability” means an explicit family of strictly disjoint readers.

is robustly public too.

*Proof.* For  $X = \bigoplus_\alpha X_\alpha \in \bigcap_k \mathcal{A}_k$ ,  $X_\alpha \in (B(\mathcal{H}_{1,\alpha}) \otimes \mathbf{1}) \cap (\mathbf{1} \otimes B(\mathcal{H}_{2,\alpha}) \otimes \mathbf{1}) = \mathbf{C}\mathbf{1}_\alpha$ .  $\square$

The content of [Theorem 4](#) is the grading assumption. Physical subsystems whose Hilbert space does not depend on  $\alpha$  (radiation, laboratories, observers) are *ungraded*:  $\alpha$  changes their dynamics, not their kinematics. In Marolf and Maxfield’s end-of-the-world (EOW) brane model the only graded observable exhibited counts the connected components of spacetime [38], which no fragment of one universe can perform. For ungraded fragments, central does not mean readable: take  $\mathcal{H} = \mathbf{C}_{F_1}^2 \otimes \mathbf{C}_{F_2}^2 \otimes \mathbf{C}_\alpha^2$ ,  $\mathcal{A}_k = B(\mathbf{C}_{F_k}^2) \otimes \mathbf{1}$ , identical sector states  $|00\rangle$  and  $H_\alpha = \alpha(X_1 + X_2)$ . The sector projections are central in the total algebra, yet  $\bigcap_k \mathcal{A}_k = \mathbf{C}\mathbf{1}$  and  $\chi(\alpha:F_k) = \chi(\alpha:\text{whole}) = 0$  at  $t = 0$ : the law is definite but unreadable.

## 6.2 Coarse-law invisibility

**Proposition 3** (Coarse-law invisibility). *Let the law  $\alpha$  consist of a coarse part  $d$  and a fine part, and let the distribution of a fragment’s state  $\rho_R(\alpha)$  given  $d$  be invariant under  $\rho \mapsto U\rho U^\dagger$  for all unitaries  $U$  of the fragment. Let  $m$  fragments in the same  $\alpha$ -sector hold  $\rho_R(\alpha)^{\otimes m}$  (the fine law is shared). Then*

- (a)  $\chi(d:R_1) = 0$  exactly;
- (b)  $\chi(d:R_1 \cdots R_m) = I(d;\lambda)$ , where the Young diagram  $\lambda$  of Schur–Weyl duality has the distribution  $\Pr(\lambda|\rho) = \dim S_\lambda s_\lambda(\text{spec } \rho)$  [39];
- (c)  $I_3(d:R_1:R_2) := I(d:R_1) + I(d:R_2) - I(d:R_1 R_2) = -\chi(d:R_1 R_2) \leq 0$ : the coarse law is purely synergistic (encoded), the opposite pole to Darwinian redundancy.

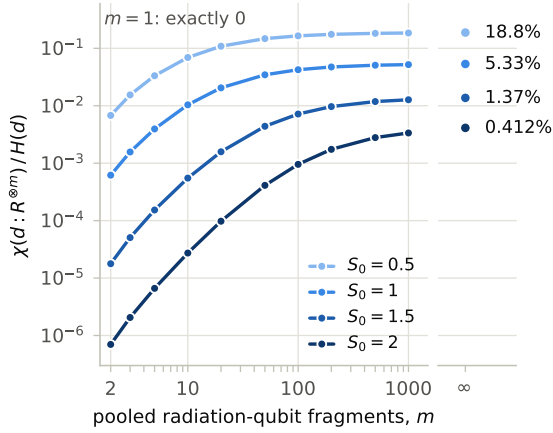
*Proof.* (a) Unitary invariance gives  $\mathbb{E}[\rho_R|d] = \mathbf{1}/k$  for every  $d$ . (b) Schur–Weyl duality writes  $\rho^{\otimes m} = \bigoplus_\lambda \pi_\lambda(\rho) \otimes \mathbf{1}_{S_\lambda}$ , and averaging over  $U$  with Schur’s lemma gives  $\mathbb{E}_U \pi_\lambda(U\rho U^\dagger) = (s_\lambda(\text{spec } \rho) / \dim U_\lambda) \mathbf{1}$ ; the averaged state is block-diagonal with  $d$  entering only through the block weights, and the dimension factors cancel in  $\chi$ . (c) follows from (a).  $\square$

In the EOW model [38] the number  $Z = d$  of boundary states is Poisson distributed with mean  $\lambda = e^{2S_0} / (1 - e^{-2S_0})$  (boundary action set to  $S_0$ , as there), and a black hole prepared in  $k$  EOW flavours leaves radiation in the normalized complex Wishart state  $\rho_R = G / \text{Tr } G$  with  $d$  degrees of freedom, which is unitarily invariant. The coarse law thus carries  $H(d) = S_0 + \frac{1}{2} \ln(2\pi e) + O(e^{-2S_0})$  nats, yet each radiation fragment holds none of it. [Figure 2](#) shows how much  $m$  pooled qubit fragments ( $k = 2$ ) recover: at most 18.8%, 5.33%, 1.37% and 0.412% of  $H(d)$  as  $m \rightarrow \infty$  at  $S_0 = 0.5, 1, 1.5, 2$  (at  $S_0 = 2$ ,  $H(d) = 3.427$  nats and the limit is 0.0141 nats). Monte Carlo computations indicate that reading the coarse law fully requires fragments of dimension  $k \gtrsim \lambda \approx e^{2S_0}$ . The fine law, by contrast, leaks weakly into single fragments: the Page deficit [40] is  $\approx 0.10$  nats at  $S_0 = 1$ .

## 6.3 A speed limit for reading laws

**Theorem 5** (Law-reading speed limit). *Let a fragment  $F$  and an ancilla start in an  $\alpha$ -independent state  $|\phi\rangle$  and evolve under  $H_{F,\alpha}(t) \otimes \mathbf{1} + C(t)$ , with controls  $C(t)$  independent of  $\alpha$  (adaptive measurements can be purified into such controls). With  $\|Y\|_{\text{sn}} := \lambda_{\max}(Y) - \lambda_{\min}(Y)$ ,*

$$\arccos |\langle \phi_a(T) | \phi_b(T) \rangle| \leq \frac{1}{2} \int_0^T \|H_{F,a} - H_{F,b}\|_{\text{sn}} dt. \quad (9)$$



**Figure 2:** The coarse law is encoded, not recorded. Readable fraction  $\chi(d:R^{\otimes m})/H(d)$  of the Marolf–Maxfield coarse law  $d$  held by  $m$  pooled radiation-qubit fragments ( $k = 2$ ), for four values of  $S_0$ ; the right panel gives the  $m \rightarrow \infty$  limits. A single fragment holds exactly zero (Proposition 3). Values are exact Schur–Weyl sums and quadratures, conditioned on  $d \geq 1$ .

Zero-error discrimination of the laws  $a$  and  $b$  therefore requires  $\int_0^T \|\Delta H_F\|_{\text{sn}} dt \geq \pi$ . The bound is attained when  $H_{F,\alpha} = H_0 + \alpha V$  with  $[H_0, V] = 0$ .

*Proof.* Let  $U_a(t)$  be the full propagator under  $a$ , controls included, and  $|\chi(t)\rangle = U_a(t)^\dagger |\phi_b(t)\rangle$ . Then  $\langle \phi_a(T) | \phi_b(T) \rangle = \langle \phi | \chi(T) \rangle$  and  $i\partial_t |\chi\rangle = K(t) |\chi\rangle$  with  $K = U_a^\dagger (H_{F,b} - H_{F,a}) U_a$ ; the common controls cancel. The Fubini–Study speed of  $|\chi\rangle$  is the standard deviation of  $K$ , at most  $\|K\|_{\text{sn}}/2 = \|\Delta H_F\|_{\text{sn}}/2$  (the Mandelstam–Tamm and Anandan–Aharonov geometry; see [41]), and the Fubini–Study distance is at most the path length. For commuting perturbations the probe  $(|v_{\text{max}}\rangle + |v_{\text{min}}\rangle)/\sqrt{2}$  attains the bound [42].  $\square$

No fragment reads a law at a glance, and a continuous coupling is never read exactly in finite time: resolution  $\varepsilon$  needs  $T \geq \pi/(\varepsilon \|\partial_\alpha H_F\|_{\text{sn}})$ . For the cosmological constant in a Newtonian static patch,  $\Phi = -\Lambda c^2 r^2/6$ , a probe of mass  $m$  spread over a distance  $L$  has  $\|\Delta H\|_{\text{sn}} = m\Lambda c^2 L^2/6$ , so  $T_* = 6\pi\hbar/(m\Lambda c^2 L^2)$ . Reading  $\Lambda$  within the age of the universe needs  $mL^2 \geq 4.6 \times 10^{-16} \text{ kg m}^2$ ; a caesium atom spread over 1 m would need about  $2 \times 10^9$  ages of the universe, and 1 kg over 1 m about 200 s. These are Heisenberg-limited lower bounds; in practice  $\Lambda$  is read from records that the cosmic expansion wrote.

## 6.4 Observers

Can observers make the law public? In Harlow’s observer rule [43] an observer with Hilbert-space dimension  $k_O = e^{S_{\text{Ob}}}$  is maximally entangled with an external clone, which carries its record. We model this in the

EOW model by letting the observer’s pointer index play the role of the EOW flavour, so that the clone’s state in the sector  $Z = d$  is the  $k_O \times k_O$  normalized Wishart matrix with  $d$  degrees of freedom. This identification is our construction: Harlow’s observer-modified inner product lives in a different, two-particle closed-universe model without  $\alpha$ -sectors.

**Proposition 4** (A single observer never holds the law). *In this model  $\chi(d:\text{Ob}') = 0$  for every  $k_O$  and every prior on  $d$ ; hence  $I_3(d:\text{Ob}'_1:\text{Ob}'_2) = -\chi(d:\text{Ob}'_1\text{Ob}'_2) \leq 0$  for any two observers, and no finite coalition of observers reads  $d$  with zero error.*

*Proof.* The first claim is Proposition 3(a), since the Gaussian entries are invariant under  $U(k_O)$ . For  $m$  observers, Howe duality gives  $\Pr(\lambda|d) = \dim U_\lambda(k_O) \dim U_\lambda(d) / \binom{k_O d + m - 1}{m}$ ; every value of  $d$  gives positive probability to the one-row diagram, so no two values are distinguished with zero error.  $\square$

Preliminary numerics<sup>†</sup> quantify what coalitions can do. Pooling infinitely many observers, the readable fraction of the coarse law ramps up from  $S_{\text{Ob}} \approx S_0$  and reaches 90% at  $S_{\text{Ob}} \approx \ln \lambda \approx 2S_0$ ; for  $S_{\text{Ob}} \gg 2S_0$  one  $\delta = 0.1$  record needs  $m_{0.1} \approx 0.87(\lambda + 2\sqrt{\lambda})^2 \approx e^{4S_0}$  observers. In Harlow’s regime  $k_O \ll e^{S_0}$ , mapped onto the EOW model as  $k_O \ll \langle Z \rangle$ , even infinite coalitions learn almost nothing. Heuristically, observers agree on the laws, by self-averaging, long before they can make them public. For an ordinary coupling constant read from ground-state blocks of the transverse-field Ising chain,  $R_0 = 0$  for every finite block, and a  $\delta$ -reader must be extensive:  $R_\delta \approx Nf/c(\delta)$ , with  $f$  the fidelity density per site [44] ( $f = |\Delta g|/8$  at criticality) and  $c(0.1) \in [1.000, 2.634]^\dagger$ .

**Corollary 2** (Baby universes; conditional). *Suppose the baby-universe Hilbert space is one-dimensional [28] and the algebra of each complete asymptotic boundary is a factor. Then for any family containing two commuting boundary fragments that generate the boundary algebra,  $\bigcap_k \mathcal{A}_k = \mathbb{C}1$  by Theorem 1(4): no fact carrying nonzero information is robustly perfectly public.*

Corollary 2 concerns state-independent publicity; state-dependent zero-error records are excluded separately, by Theorem 3 in bulk effective field theory.

## 7 Discussion

**Privacy and publicity as two faces of “no global symmetries”.** Table 3 summarizes where perfect records can live. A 0-form global symmetry is exactly what allows a charge to be perfectly private (Proposition 2); an exact higher-form symmetry is what allows it to be perfectly public in bounded regions (Theorem 2), and

| Fact                                | Zero-error readers in bounded regions             | Where exact records live                                               | Reference                     |
|-------------------------------------|---------------------------------------------------|------------------------------------------------------------------------|-------------------------------|
| charge of a global 0-form symmetry  | none outside the carrier (perfectly private)      | nowhere outside; excluded in quantum gravity                           | <a href="#">Proposition 2</a> |
| gauge charge, exact 1-form symmetry | every linking shell ( $R_0 \geq \nu$ , unbounded) | every linking shell                                                    | <a href="#">Theorem 2</a>     |
| gauge charge, dynamical carriers    | none ( $R_0 = 0$ , faithful states)               | spatial infinity only                                                  | <a href="#">Theorem 3</a>     |
| Poincaré charges (mass, ...)        | conjectured none; $R_\delta \gtrsim \sqrt{r}$     | spatial infinity (ADM)                                                 | <a href="#">Section 5</a>     |
| $\alpha$ -label, graded fragments   | every fragment                                    | every fragment (kinematic)                                             | <a href="#">Theorem 4</a>     |
| $\alpha$ -label, physical fragments | none; $\chi = 0$ per fragment                     | in no physical fragment (encoded)                                      | <a href="#">Proposition 3</a> |
| coupling constants                  | none in a snapshot                                | time-extended readers, after $T \geq \pi / \ \Delta H_F\ _{\text{sn}}$ | <a href="#">Theorem 5</a>     |

**Table 3:** Where perfect records live. Granting no global symmetries, completeness ([Corollary 1](#)) and McNamara–Vafa ([Corollary 2](#)), the global-symmetry, 1-form-symmetry and  $\alpha$ -label rows do not arise in quantum gravity, and exact records of charges live only at infinity.

without one bounded records are never exact ([Theorem 3](#) and [Proposition 1](#)), up to the open case of locally superselected charges without a higher-form symmetry. Quantum gravity is believed to forbid both kinds of symmetry, and the two prohibitions act in opposite directions: removing 0-form symmetries forces charges to be announced, and removing 1-form symmetries makes every announcement in a bounded region imperfect. “No global symmetries of any degree” then reads: every conserved charge is announced with zero error at infinity, and every record has a local writer.

**Strong-to-weak symmetry breaking.** For a 0-form symmetry, a charge that is encoded rather than recorded, not readable by any half of a system below a threshold charge difference, is closely tied to strong-to-weak spontaneous symmetry breaking (SW-SSB); this connection is already made in the SW-SSB literature [[45](#), [46](#)]. For 1-form symmetries, the toric code with independent errors ([Section 5](#)) shows the corresponding transition in redundancy,  $R_\delta \propto r / \ln r$  below the decoding threshold and  $O(1)$  above it<sup>†</sup>, and that threshold has been identified with a mixed-state topological transition and with strong-to-weak breaking of a 1-form symmetry [[47](#), [48](#)]. We conjecture that the redundancy of a charge is an order parameter for an exact or emergent 1-form symmetry:  $R_0 = \nu$  when it is exact,  $R_\delta$  unbounded when it is broken explicitly by gapped carriers, and  $R_\delta = O(1)$  when it is lost. Preliminary exact results for a one-dimensional gauge–Higgs chain support this<sup>†</sup>.

**Generalized symmetries.** Casini and Magán showed that completeness in AQFT, the maximality of the local algebras, is equivalent to the absence of generalized symmetries [[23](#), [24](#)], which entropic order parameters detect [[25](#)]. [Theorems 2](#) and [3](#) give this dichotomy an

operational meaning in terms of records: with a generalized symmetry, topological operators are perfect public records on every linking shell; without one, every bounded record is noisy, with noise set by the lightest carriers. The link between electric 1-form symmetry breaking and the weak gravity conjecture [[49](#), [50](#)] suggests a cap  $R_\delta \lesssim g q M_{\text{Pl}} r$  on gauge-charge redundancy, about  $5 \times 10^{33}$  per metre for QED; this is heuristic, since it needs the gapped scaling  $R \sim mr$  to be an upper bound.

**Holography.** In AdS/CFT the boundary theory is non-gravitational and can have exact higher-form symmetries;  $SU(N)$   $\mathcal{N} = 4$  super-Yang–Mills, for example, has a  $\mathbb{Z}_N$  1-form symmetry [[20](#), [51](#)]. Boundary line charges are then perfectly public on boundary shells. This is not a counterexample to [Corollary 1](#): the boundary is the bulk’s conformal infinity, where our results place exact records. Bulk bounded regions keep  $R_0 = 0$ , with dynamical strings as carriers (a step we have only sketched), and bulk gauge charges are dual to boundary 0-form charges, whose homological packing number is at most one ([Theorem 2\(b\)](#); see [[26](#)] for the dictionary). Whether the boundary is in a precise sense more public than the bulk is under investigation.

**Open problems.** (1) Is  $\ln r$  optimal for massless carriers, and  $\sqrt{r}$  for mass, over arbitrary measurements? (2) Is there a theory with locally superselected charges but no higher-form symmetry? It would evade [Theorem 3](#). (3) Prove local normality in continuum QED with its infrared structure, and formulate it for Poincaré charges with gravitational dressing [[33](#)]. (4) Find a quantitative privacy–publicity trade-off; only the privacy side (5) is known. (5) Restate the results for dressed, relational observables, where compact-region algebras may not exist [[28](#)]. (6) Establish or refute the weak-gravity

cap, and a complementarity of publicity for dual charges with a mixed anomaly. (7) For laws: observers inside one universe, time-extended observers, and a proof that coalitions need  $m_\delta \propto \lambda^2$  observers.

In a universe with gravity and a complete charge spectrum, every conserved charge leaves a perfect record, but only at infinity. No finite region ever holds a perfect copy, and none can acquire one in finite time. Near-perfect copies, however, are everywhere.

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